11. The management of a factory intended to maximize the gross margin (expressed in *m. u.*) with the production of two products, **P1** and **P2**. With that purpose formulated the following LP problem:

$$Max Z = 20x_1 + 10x_2$$

s.t.:

$$\begin{cases} 3x_1 + x_2 \le 18 & \text{section 1 (m.h.)} \\ x_1 + x_2 \le 10 & \text{section 2 (m.h.)} \\ 2x_1 + 5x_2 \ge 20 & \text{market requirements} \\ x_1, x_2 \ge 0 \end{cases}$$

Solve the problem and write a short report to the factory management, and underline the most relevant aspects of the production plan to be followed.

12. Consider the following LP problem:

$$Max Z = 3x_1 + 2x_2 \quad \text{(total profit, in } m.u.\text{)}$$

$$\text{s.t.:} \begin{cases} x_1 \le 4 & \text{(resource 1)} \\ x_1 + 3x_2 \le 15 & \text{(resource 2)} \\ 2x_1 + x_2 \le 10 & \text{(resource 3)} \\ x_1, x_2 \ge 0 \end{cases}$$

a) Solve the problem by graphical method, by the Simplex algorithm and by the *Solver/Excel*.

b) Write the dual of the above formulated problem.

c) Find the optimal solution of the dual with the help of the graphical solution of the primal, reading the output reports of *Solver* and solving the dual itself by the *Solver*.

d) Assuming that it is a problem to find the level at which activities, that share limited resources, should be performed, explain the economic meaning of the optimal solutions.

e) Determine the impact in the total profit of a reduction of the availability of resource 3 to 8 units.

13. Write the dual associated to the following LP problem:

$$Max Z = 6x_1 + 8x_2$$

$$\text{s.a:} \begin{cases} 5x_1 + 2x_2 \le 20 \\ x_1 + 2x_2 \le 10 \\ x_1, x_2 \ge 0 \end{cases}$$

a) Solve by graphical method the pair of dual problems.

b) Solve the primal problem by an algorithm. Write and classify the solution associated to each simplex tableaux and identify them in the graphic.

14. Consider the following LP problem:

$$Max Z = 2x_1 + 7x_2 + 4x_3$$

$$\text{s.t.: } \begin{cases} x_1 + 2x_2 + x_3 \le 10 \\ 3x_1 + 3x_2 + 2x_3 \le 10 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

a) Write its dual.

b) Solve both problems.

15. Consider the following LP problem:

$$Min Z = x_1 + 3x_2$$

$$\text{s.t.: } \begin{cases} x_1 + x_2 \ge 4 \\ -x_1 + x_2 \ge 0 \\ -x_2 \ge -6 \\ x_1, x_2 \ge 0 \end{cases}$$

a) Solve it by graphical method and write the dual problem associated with this LP problem.

b) Solve the given problem by *Solver/Excel* and display the solutions of both problems.

16. A firm wants to study the future production plan of products **P1**, **P2** and **P3**. In order to maximize the global profit, the following LP problem was formulated:

$$Max Z = 3x_1 + 4x_2 + 2x_3$$

$$\text{s.t.: } \begin{cases} x_1 + x_2 + 2x_3 \le 10 \\ 2x_1 + 4x_2 + x_3 \le 8 \\ 2x_1 + 3x_2 + 2x_3 \le 20 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

where x_j represents the quantity of product **Pj**, $j = 1, 2, 3$ that should be produced;

the first two constraints refers to the consumption of raw materials **rm1** and **rm2**, respectively;

the third constraint is associated to the limited availability of store space in the warehouse.

Assume that the optimal production plan indicates that only *2* units of **P1** and *4* units **P3** should be produced.

a) Without solving the problem but with the information that the first shadow-price is *1/3*, determine the internal values of the resources (raw materials and store space) and give the economic interpretation of those values.

b) Obtain the output *Solver/Excel* reports and find the increase in the actual unit profit of **P2** in order that it becomes advantageous to include it in the production plan.

17. Consider the following LP problem:

$$MinZ = 3x_1 + 2x_2$$

$$\text{s.t:} \begin{cases} x_1 \leq 3 \\ 3x_2 \leq 12 \\ \alpha x_1 + x_2 \geq 6 \\ x_1,\ x_2 \geq 0 \end{cases}$$

a) Take $\alpha = 1$ and solve the problem. Write the dual and solve it.

b) Find a value for α so that alternative optimal solutions can be found.

c) Find a value for α so that the problem is infeasible.

18. Consider again exercise **11**.

a) Given the production plan found, and in order to face problems in selling product 1 the factory's management contacted the two companies. Company **A** proposal is to acquire the entire production of both products but wants a discount of 30 *m.u.*; company **B** only accepts to buy **P1**'s production, but wants to negotiate its unit price. Describe how negotiations should be conducted, and the contract you would sign.

b) How much would the factory management willing to pay for a change that would provide an increase of 6 m.h. in section 1 and a reduction of 2 m.h. in section 2? Explain, assuming that the initial resources of the two sections were acquired at no cost.

19. Consider the following LP problem, formulated by the OR department of a company, which intends to optimize the total monthly revenue from the sale of four products (**P1**, **P2**, **P3** and **P4**), produced using two types of raw material (**rm1** and **rm2**):

$$MaxZ = 2x_1 + 3x_2 + 7x_3 + 4x_4$$

$$\text{s.t.:} \begin{cases} x_1 + x_2 + x_3 + x_4 \leq 9 & (\mathbf{rm1}) \\ x_1 + 2x_2 + 4x_3 + 8x_4 \leq 24 & (\mathbf{rm2}) \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

Considering the solution of the problem, the department informs that only products **P1** and **P3** should be produced.

a) Which of the two products not included in the production plan, would need a smaller increase in the unit selling price in order that its production turns to be profitable?

b) Assume that due to difficulties on import, next month only 5 units of **rm1** will be available. Determine the new optimal production plan and the total revenue associated to it.

c) Suppose that the company can produce a new product, with a unit selling price of 10 *m.u.*, and a need of 2 units from each one of the raw materials to produce one unit of the new product. What should be the new production plan?

d) Suppose that a budged of 24 *m.u.* is now available to spend in only one of the raw materials. Let 4 *m.u.* and 8 *m.u.* be the unit price to acquire each extra unit of that raw materials, respectively. Which decision should be made if the companies' management pretends keep the production of **P1** and **P3** only? What are the consequences for the company?

20. Consider the *output* of *Solver/Excel* of the following problem, where x_1, x_2 and x_3 are the quantities sold of products 1, 2 and 3, respectively:

$$Min\ Z = 2x_1 + 3x_2 + 8x_3 \quad \text{(total cost)}$$

$$\text{s.t.:} \begin{cases} x_1 + x_2 + x_3 \geq 90 & \text{(minimu quantity)} \\ 5x_1 + 4x_2 + 3x_3 \leq 400 & \text{(maximum capacity)} \\ 3x_1 - 5x_2 = 0 & \text{(relation between sells of products 1 and 2)} \\ x_1,\ x_2,\ x_3 \geq 0 & \end{cases}$$

a) Display the optimal solution of the primal and interpret the meaning of all its variables (including slack variables).

b) Display and interpret the shadow prices.

c) What are the consequences in the total cost if the third constraint changes to $3x_1 - 5x_2 = 100$?

d) Suppose that the unit cost of product 3 is now 4.5, determine the consequences in the selling plan as in the total cost.

21. In factory *Choco* three new types chocolate bars are going to be made for the food industry. Each bar is made of sugar and chocolate only.

Bar	quantity of sugar (*kg/bar*)	quantity of chocolate (*kg/bar*)	profit of each chocolate bar (*m.u.*)
Type 1	1	2	3
Type 2	1	3	7
Type 3	1	1	5
availabilities (*kg*)	50	100	

To formulate the problem we define variables x_j, representing the number of chocolate bars **Type j** to make, where j=1,2,3.

Answer to the following questions using, when needed, the *Solver/Excel* to find the solution of the LP problems.

a) For which profit values of chocolate bars of **Type 2** does the current solution remains optimal? Which will be the optimal solution in case the profit is 13 *m.u.*?

b) Is it worth considering an increase in the availability of sugar?

c) For which amount of sugar is the set of basic variables in the optimal solution the same?

d) Is it worth considering an increase in the availability of chocolate?

e) For which amount of chocolate is the set of basic variables in the optimal solution the same?

f) If the amount of sugar available was of 60 *kg*, which would be the total profit of these products? Which should be the production plan that *Choco* should apply in these conditions?

g) Repeat the previous question for an availability of sugar of 40 *kg* and of 30 *kg*.

h) Two new types of chocolate bars are being considered for production in *Choco*. One of the types, **Type 4**, needs 1 *kg* of sugar and 4 *kg* of chocolate per bar, providing a unit profit of 11 *m.u.*. For the other type of bar, **Type 5**, 2 *kg* of sugar and 1 *kg* of chocolate per bar are needed, for a unit profit of 13 *m.u.*. Make a decision and explain it.

i) Which values can the unit profit on chocolate bars of **Type 1** have, so that the current solution is maintained optimal? Which will be the optimal solution if that profit is 7 *m.u.*?

22. An humanitarian organization intends to plan a medicaments distribution program in two regions located in the Great Lakes area of Africa. For strategic and security purposes it is possible to use 3 airports from which, by land routes, the supply of the two regions will take place. Considering that the transportation cost of medicaments to the airports should be minimized, insuring, in each of the two regions, a minimum number of people is contemplated by the program, the following LP model has been formulated:

$$Min\,Z = 40x_1 + 18x_2 + 30x_3 \qquad \text{(in } m.u.\text{)}$$

$$\text{s.a:}\begin{cases} 4x_1 + x_2 + x_3 \geq 250 & \text{(thousands of people)} \\ 4x_1 + 3x_2 + 6x_3 \geq 350 & \text{(thousands of people)} \\ x_1,\ x_2,\ x_3 \geq 0 \end{cases}$$

where $x_j =$ tones of medicaments to be shipped to airport *j* (*j*=1,2,3).

Obtain the optimal solution for the problem by *Solver/Excel*.

a) Write short report presenting the problem's solution, referring the value of the dual decision variables as well as its meaning.

b) If the number of thousands of people to be contemplated in the first region is 350 instead of 250, which will be the new cost for the program?

c) Determine the changes in the solution if airport 1 cannot receive more than 40 *ton.* of medicaments.

23. Solve the following LP problem referring to the production of **P1**, **P2** e **P3,** using *Solver/Excel*:

$$Max\,Z = 10x_1 + 20x_2 + 15x_3$$

$$\text{s.a:}\begin{cases} x_1 + 3x_2 + 2x_3 \leq 80 \\ 4x_1 + 10x_2 + 5x_3 \leq 90 \\ 4x_1 + 10x_2 \geq 50 \\ x_1,\ x_2,\ x_3 \geq 0 \end{cases}$$

The objective function refers to the maximization of the total revenue, and the first and the second constraints are related to the machine hours available in sections 1 and 2, respectively, and the third constraint determines the minimum financial margin that should be achieved. The financial margin is the difference between the total revenue and the total variable costs.

a) Write and interpret the optimal solutions for the primal and dual problems.

b) The revenue of **P2** just increased 20%, although its financial margin is maintained. Indicates which are the arising changes of the production program and the shadow-prices.

c) How much should the revenue of **P2** increase (maintaining its financial margin) so that this product is included in the production plan?

d) Indicate a way of increasing the revenue by at least 2% through the changes in the available quantities of the company´s resources.

e) Which are the changes in the optimal problem arising from the new market requirements which obligates a minimum production for **P2** of 4 units? Same question for **P1**.

24. Consider the following LP problem:

$$Min\ Z = x_1$$

$$\text{s.to: } \begin{cases} 2x_1 + 2x_2 \geq 10 \\ -x_1 + \ x_2 \leq 5 \\ 3x_1 + 2x_2 \leq 30 \\ 2x_1 - \ x_2 \leq 16 \\ x_1, x_2 \geq 0 \end{cases}$$

a) Solve it by a graphical method.

b) Perform the sensitivity analysis to the right hand side of the third functional constraint graphically.

c) Without solving the dual, determine the optimal value of the dual variable associated to the third functional constraint.

d) Without using the graphic (nor solving the new problem) check if the introduction of the constraint $x_1 + x_2 \geq 3$ changes the feasible region.